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COMPARISON OF WAVE-MODE COORDINATE AND PULSE SUMMATION
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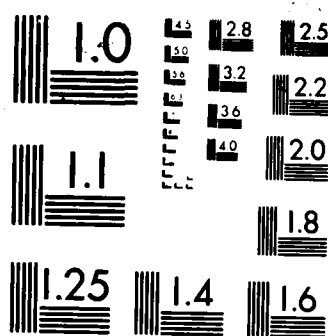
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Nondispersive pulse propagation in a simple one-dimensional lattice structure is analyzed using the pulse summation method and the wave-mode coordinate method. The results of the two methods are shown to be identical, and both methods account for the existence of equivalent paths in the lattice. Some recommendations for future research are given.

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INTRODUCTION

Pulse propagation and wave propagation comprise an important class of problems in the study of the dynamics of large lattice structures. The study of pulse and wave propagation has applications in dynamic failure, control, and nondestructive evaluation.

In this investigation, nondispersive pulse propagation in a simple one-dimensional lattice structure is analyzed, using both the pulse summation method and the wave-mode coordinate method. It is shown that the pulse summation method (a time domain method) and the wave-mode coordinate method (a frequency domain method) give identical results, and that both methods account for the existence of equivalent paths in the lattice structure. Some examples of equivalent paths are given. Also, some recommendations are made for possible extensions of the analysis presented here.

ANALYSIS

Lattice Definition and Problem Statement

A one-dimensional lattice structure consisting of two segments and one joint is shown in Fig. 1. It is assumed that the joint is rigid and massless, and that the extent of the joint in the x-direction is small in comparison with ℓ_1 or ℓ_2 . Segment one has elastic modulus E_1 , cross-sectional area A_1 , and mass density ρ_1 . Segment two has elastic modulus E_2 , cross-sectional area A_2 , and mass density ρ_2 . It is assumed that longitudinal wave propagation in each segment is governed by the classical one-dimensional wave equation. Therefore, disturbances in longitudinal force or longitudinal displacement propagate nondispersively in each segment, and a longitudinal force or displacement pulse introduced into either segment will maintain its shape as it propagates. The velocity of pulse propagation in segment one is

$$c_1 = \sqrt{\frac{E_1}{\rho_1}} \quad (1)$$

and the velocity of pulse propagation in segment two is

$$c_2 = \sqrt{\frac{E_2}{\rho_2}} \quad (2)$$

The characteristic transit time required for a pulse to traverse the length of segment one is

$$\tau_1 = \frac{\ell_1}{c_1} \quad (3)$$

and the characteristic transit time required for a pulse to traverse the length of segment two is

$$\tau_2 = \frac{\ell_2}{c_2} \quad (4)$$

The lengths ℓ_1 and ℓ_2 are defined in Fig. 1. It is assumed here that the characteristic transit time for segment one is equal to the characteristic transit time for segment two, or

$$\tau_1 = \tau_2 = \tau \quad (5)$$

The problem considered here is the following. A longitudinal force $\mathcal{F}(t)$ is applied to the joint as shown in Fig. 1. It is assumed that the force $\mathcal{F}(t)$ is an impulse of the form

$$\mathcal{F}(t) = \mathcal{F}_0 \delta(t) \quad (6)$$

It is desired to find the resulting longitudinal force $F_1(t)$ at point 1, the left-hand end of segment one. Note that if point 1 is a free end, then $F_1(t) = 0$, directly from the boundary condition at a free end.

Pulse Summation Solution

Using the pulse summation method described in [1], the following solution may be obtained for $F_1(t)$:

$$F_1(t) = \sum_{n=0}^{\infty} \mathcal{F}_0 \left(\frac{R_1}{R_1 + R_2} \right) (1 - r_0) \left[S(n) + \sum_{K_1=1}^{n-1} \sum_{L_1=1}^{\min(K_1, K_2)} N_1(K_1, K_2, L_1) S_1(K_1, K_2, L_1) \right. \\ \left. + \sum_{K_2=1}^n \sum_{L_1=0}^{\min(K_1, K_2-1)} N_2(K_1, K_2, L_1) S_2(K_1, K_2, L_1) \right] \cdot \delta(t - (2n + 1)\tau) \quad (7)$$

where

$$K_1 + K_2 = n \quad (8)$$

$$S(n) = (r_0 r_1)^n \quad (9)$$

$$S_1(K_1, K_2, L_1) = r_0^{K_1} r_1^{K_1 - L_1} t_1^{L_1} t_2^{L_1} r_2^{K_2 - L_1} r_3^{K_2} \quad (10)$$

$$N_1(K_1, K_2, L_1) = \binom{K_1}{L_1} \binom{K_2 - 1}{L_1 - 1} \quad (11)$$

$$S_2(K_1, K_2, L_1) = r_0^{K_1} r_1^{K_1 - L_1} t_1^{L_1 + 1} t_2^{L_1} r_2^{K_2 - L_1 - 1} r_3^{K_2} \quad (12)$$

$$N_2(K_1, K_2, L_1) = \binom{K_1}{L_1} \binom{K_2 - 1}{L_1} \quad (13)$$

$$r_1 = \frac{R_1 - R_2}{R_1 + R_2} \quad (14)$$

$$r_2 = \frac{R_2 - R_1}{R_1 + R_2} \quad (15)$$

$$t_1 = \frac{2R_2}{R_1 + R_2} \quad (16)$$

$$t_2 = \frac{2R_1}{R_1 + R_2} \quad (17)$$

$$R_1 = A_1 \sqrt{\rho_1 E_1} \quad (18)$$

$$R_2 = A_2 \sqrt{\rho_2 E_2} \quad (19)$$

The quantities r_1 and r_2 are the (displacement) reflection coefficients at the joint, and the quantities t_1 and t_2 are the (displacement) transmission coefficients at the joint. The coefficient r_1 is the reflection coefficient for a pulse which arrives at the joint from segment 1, and the coefficient r_2 is the reflection coefficient for a pulse which arrives at the joint from segment 2. The coefficient t_1 is the transmission coefficient for a pulse entering segment 1 from segment 2, and the coefficient t_2 is the transmission coefficient for a pulse entering segment 2 from segment 1. The quantities r_0 and r_3 are the (displacement) reflection coefficients at the left-hand boundary of the lattice (point 1) and the right-hand boundary of the lattice (point 4), respectively. The reflection coefficient r_0 may be determined from the boundary conditions at point 1. If, for example, point 1 is a fixed end, $r_0 = -1$, and if point 1 is a free end, $r_0 = 1$. Similarly, the reflection coefficient r_3 may be determined from the boundary conditions at point 4.

The methods used in the derivation of eqn. (7) are discussed in detail in [1]. Eqn. (7) is a slightly corrected form of eqn. (A90) in [1], and it corresponds to the sum of cases I and III defined in Appendix A of [1]. Eqn. (7) consists of an infinite series of impulses which are delayed by odd multiples of the characteristic transit time τ . The quantities S , S_1 and S_2 , which contribute to the amplitudes of the impulses, consist of powers of the reflection and transmission coefficients. The quantities N_1 and N_2 are numerical coefficients which will be discussed and interpreted subsequently. Writing out the first few terms of eqn. (7) gives

$$F_1(t) = \mathcal{F}_0 \left(\frac{R_1}{R_1 + R_2} \right) (1 - r_0) \\ \cdot \left[\delta(t - \tau)[1] + \delta(t - 3\tau)[r_0 r_1 + t_1 r_3] \right]$$

$$\begin{aligned}
& + \delta(t - 5\tau) \left[r_0^2 r_1^2 + r_0 t_1 t_2 r_3 + r_0 r_1 t_1 r_3 + t_1 r_2 r_3^2 \right] \\
& + \delta(t - 7\tau) \left[r_0^3 r_1^3 + r_0 t_1 t_2 r_2 r_3^2 + 2r_0^2 r_1 t_1 t_2 r_3 \right. \\
& \quad \left. + r_0^2 r_1^2 t_1 r_3 + r_0 r_1 t_1 r_2 r_3^2 + r_0 t_1^2 t_2 r_3^2 + t_1 r_2^2 r_3^3 \right] \\
& + \delta(t - 9\tau) \left[r_0^4 r_1^4 + r_0 t_1 t_2 r_2^2 r_3^3 + 2r_0^2 r_1 t_1 t_2 r_2 r_3^2 + r_0^2 t_1^2 t_2^2 r_3^2 \right. \\
& \quad \left. + 3r_0^3 r_1^2 t_1 t_2 r_3 + r_0^3 r_1^3 t_1 r_3 + r_0^2 r_1^2 t_1 r_2 r_3^2 + 2r_0^2 r_1 t_1^2 t_2 r_3^2 \right. \\
& \quad \left. + r_0 r_1 t_1^2 r_2^2 r_3^3 + 2r_0 t_1^2 t_2 r_2 r_3^3 + t_1 r_2^3 r_3^4 \right] \\
& + \delta(t - 11\tau) \left[r_0^5 r_1^5 + r_0 t_1 t_2 r_2^3 r_3^4 + 2r_0^2 t_1^2 t_2^2 r_2 r_3^3 + 2r_0^2 r_1 t_1 t_2 r_2^2 r_3^3 \right. \\
& \quad \left. + 3r_0^3 r_1^2 t_1 t_2 r_2 r_3^2 + 3r_0^3 r_1 t_1^2 t_2^2 r_3^2 + 4r_0^4 r_1^3 t_1 t_2 r_3 + r_0^4 r_1^4 t_1 r_3 \right. \\
& \quad \left. + r_0^3 r_1^3 t_1 r_2 r_3^2 + 3r_0^3 r_1^2 t_1 t_2 r_3^2 + r_0^2 r_1^2 t_1 r_2^2 r_3^3 + \underline{4r_0^2 r_1 t_1^2 t_2 r_2 r_3^3} \right. \\
& \quad \left. + r_0^2 t_1^3 t_2^2 r_3^3 + r_0 r_1 t_1^3 r_2^3 r_3^4 + 3r_0 t_1^2 t_2^2 r_2^2 r_3^4 + t_1 r_2^4 r_3^5 \right] \\
& + \dots \Big|
\end{aligned} \tag{20}$$

Wave-Mode Coordinate Solution

Using the wave-mode coordinate method described in [2], the following solution may be obtained for $F_1(t)$:

$$\begin{aligned}
F_1(t) = & \mathcal{F}_0 \left(\frac{R_1}{R_1 + R_2} \right) (1 - r_0) \\
& \cdot \delta(t) \left\{ \sum_{n=0}^{\infty} (r_0 r_1)^n \lambda((2n+1)\tau) \right. \\
& + \sum_{n=1}^{\infty} (r_0 t_2 r_3 t_1)^n \left(\sum_{m=0}^{\infty} (r_0 r_1)^m P(n+1, m) \lambda(2n\tau) \lambda((2m+1)\tau) \right) \\
& \cdot \left(\sum_{m=0}^{\infty} (r_2 r_3)^m P(n, m) \lambda(2n\tau) \lambda(2m\tau) \right) \\
& + \sum_{n=0}^{\infty} r_3 t_1 (r_0 t_2 r_3 t_1)^n \left(\sum_{m=0}^{\infty} (r_0 r_1)^m P(n+1, m) \lambda(2n\tau) \lambda((2m+1)\tau) \right) \\
& \cdot \left(\sum_{m=0}^{\infty} (r_2 r_3)^m P(n+1, m) \lambda(2n\tau) \lambda(2(m+1)\tau) \right) \Bigg\}
\end{aligned} \tag{21}$$

where

$$P(n, m) = \frac{(n + m - 1)!}{(n - 1)! m!} \tag{22}$$

and $\lambda(\tau)$ is a time-shift operator defined by

$$f(t)\lambda(\tau) = f(t - \tau) \tag{23}$$

The quantities r_1 , r_2 , t_1 and t_2 are defined by eqns. (14) through (17), and r_0 and r_3 are again the reflection coefficients at the left-hand and right-hand boundaries of the lattice, respectively.

The derivation of eqn. (21) is discussed in detail in [2]. Eqn. (21) may be obtained by setting $\tau_1 = \tau_2 = \tau$ (where τ_1 and τ_2 are the

characteristic transit times defined previously) in eqn. (103) of [2].

Writing out the first few terms of eqn. (21) gives

$$\begin{aligned}
 F_1(t) = & \mathcal{F}_0 \left(\frac{R_1}{R_1 + R_2} \right) (1 - r_0) \\
 & \cdot \left\{ \delta(t) \left[\lambda(\tau) + r_0 r_1 \lambda(3\tau) + (r_0 r_1)^2 \lambda(5\tau) + \dots \right] \right. \\
 & + r_3 t_1 \delta(t) \left[\lambda(\tau) + r_0 r_1 \lambda(3\tau) + (r_0 r_1)^2 \lambda(5\tau) + \dots \right] \\
 & \quad \cdot \left[\lambda(2\tau) + r_2 r_3 \lambda(4\tau) + (r_2 r_3)^2 \lambda(6\tau) + \dots \right] \\
 & + (r_0 t_2 r_3 t_1) \delta(t) \left[\lambda(3\tau) + 2r_0 r_1 \lambda(5\tau) + 3(r_0 r_1)^2 \lambda(7\tau) + \dots \right] \\
 & \quad \cdot \left[\lambda(2\tau) + r_2 r_3 \lambda(4\tau) + (r_2 r_3)^2 \lambda(6\tau) + \dots \right] \\
 & + r_3 t_1 (r_0 t_2 r_3 t_1) \delta(t) \left[\lambda(3\tau) + 2r_0 r_1 \lambda(5\tau) + 3(r_0 r_1)^2 \lambda(7\tau) + \dots \right] \\
 & \quad \cdot \left[\lambda(4\tau) + 2r_2 r_3 \lambda(6\tau) + 3(r_2 r_3)^2 \lambda(8\tau) + \dots \right] \\
 & + (r_0 t_2 r_3 t_1)^2 \delta(t) \left[\lambda(5\tau) + 3r_0 r_1 \lambda(7\tau) + 6(r_0 r_1)^2 \lambda(9\tau) + \dots \right] \\
 & \quad \cdot \left[\lambda(4\tau) + 2r_2 r_3 \lambda(6\tau) + 3(r_2 r_3)^2 \lambda(8\tau) + \dots \right] \\
 & + r_3 t_1 (r_0 t_2 r_3 t_1)^2 \delta(t) \left[\lambda(5\tau) + 3r_0 r_1 \lambda(7\tau) + 6(r_0 r_1)^2 \lambda(9\tau) + \dots \right] \\
 & \quad \cdot \left[\lambda(6\tau) + 3r_2 r_3 \lambda(8\tau) + 6(r_2 r_3)^2 \lambda(10\tau) + \dots \right] \\
 & + \dots \left. \right\}
 \end{aligned}
 \tag{24}$$

Grouping the terms in eqn. (24) according to their time delays gives

$$\begin{aligned}
 F_1(t) = & \mathcal{F}_0 \left(\frac{R_1}{R_1 + R_2} \right) (1 - r_0) \\
 & \cdot \left\{ \delta(t)\lambda(\tau)[1] + \delta(t)\lambda(3\tau) \left[r_0 r_1 + r_3 t_1 \right] \right. \\
 & + \delta(t)\lambda(5\tau) \left[(r_0 r_1)^2 + r_3 t_1 r_0 r_1 + r_3 t_1 r_2 r_3 + r_0 t_2 r_3 t_1 \right] \\
 & + \delta(t)\lambda(7\tau) \left[(r_0 r_1)^3 + r_3 t_1 (r_2 r_3)^2 + r_3 t_1 r_0 r_1 r_2 r_3 \right. \\
 & \quad \left. + r_3 t_1 (r_0 r_1)^2 + (r_0 t_2 r_3 t_1) r_2 r_3 \right. \\
 & \quad \left. + 2(r_0 t_2 r_3 t_1) r_0 r_1 + r_3 t_1 (r_0 t_2 r_3 t_1) \right] \\
 & + \delta(t)\lambda(9\tau) \left[(r_0 r_1)^4 + r_3 t_1 (r_2 r_3)^3 + r_3 t_1 r_0 r_1 (r_2 r_3)^2 \right. \\
 & \quad \left. + r_3 t_1 (r_0 r_1)^2 r_2 r_3 + r_3 t_1 (r_0 r_1)^3 \right. \\
 & \quad \left. + r_0 t_2 r_3 t_1 (r_2 r_3)^2 + 2r_0 t_2 r_3 t_1 r_0 r_1 r_2 r_3 \right. \\
 & \quad \left. + 3r_0 t_2 r_3 t_1 (r_0 r_1)^2 + 2r_3 t_1 r_0 t_2 r_3 t_1 r_2 r_3 \right. \\
 & \quad \left. + 2r_3 t_1 r_0 t_2 r_3 t_1 r_0 r_1 + (r_0 t_2 r_3 t_1)^2 \right] \\
 & + \delta(t)\lambda(11\tau) \left[(r_0 r_1)^5 + r_3 t_1 (r_2 r_3)^4 + r_3 t_1 r_0 r_1 (r_2 r_3)^3 \right. \\
 & \quad \left. + r_3 t_1 (r_0 r_1)^2 (r_2 r_3)^2 + r_3 t_1 (r_0 r_1)^3 r_2 r_3 \right]
 \end{aligned}$$

$$\begin{aligned}
& + r_3 t_1 (r_0 r_1)^4 + r_0 t_2 r_3 t_1 (r_2 r_3)^3 \\
& + 2 r_0 t_2 r_3 t_1 r_0 r_1 (r_2 r_3)^2 \\
& + 3 r_0 t_2 r_3 t_1 (r_0 r_1)^2 r_2 r_3 \\
& + 4 r_0 t_2 r_3 t_1 (r_0 r_1)^3 \\
& + 3 r_3 t_1 (r_0 t_2 r_3 t_1) (r_2 r_3)^2 \\
& + \underline{4 r_3 t_1 (r_0 t_2 r_3 t_1) r_0 r_1 r_2 r_3} \\
& + 3 r_3 t_1 (r_0 t_2 r_3 t_1) (r_0 r_1)^2 \\
& + 2 (r_0 t_2 r_3 t_1)^2 r_2 r_3 \\
& + 3 (r_0 t_2 r_3 t_1)^2 r_0 r_1 \\
& + r_3 t_1 (r_0 t_2 r_3 t_1)^2 \Big] \\
& + \dots \Big\}
\end{aligned} \tag{25}$$

Comparison of Pulse Summation Solution and Wave-Mode Coordinate Solution

Since both eqn. (7) and eqn. (21) are expressions for the same physical quantity $F_1(t)$, eqns. (7) and (21) give, when expanded, identical results. Note that if point 1 is a free end, then $r_0 = 1$, and both eqns. (7) and (21) give $F_1(t) = 0$, as required by the boundary condition at a free end. It can be seen from eqns. (20) and (25) that the first few

terms of eqn. (7) are indeed identical with the corresponding terms of eqn. (21). Note that in the pulse summation solution given by eqn. (7), the impulses are grouped according to the time delay, whereas in the wave-mode coordinate solution given by eqn. (21) some manipulation is necessary before the impulses can be grouped according to the time delay.

Equivalent Paths

The numerical constants represented by the quantities $N_1(K_1, K_2, L_1)$ and $N_2(K_1, K_2, L_1)$ in eqn. (7) and the quantities $P(n+1, m)$ and $P(n, m)$ in eqn. (21) are due to the existence of equivalent paths from the input location (the joint) to the output location (point 1) of the lattice in Fig. 1. The concept of equivalent paths in one-dimensional structures consisting of multiple segments is discussed in detail in [1]. Basically, a path A from input location to output location is equivalent to a path B from input location to output location if a pulse which travels along path A arrives at the output location with the same time delay and the same amplitude as an identical pulse which travels along path B. For example, the four equivalent paths represented by the underlined term $4r_0^2 r_1^2 t_1^2 t_2^2 r_2^3$ in eqns. (20) and (25) are shown on x-t diagrams in Fig. 2. (The coordinate x is defined in Fig. 1.) An impulse of initial amplitude \mathcal{F}_0 arrives, after following any of the four paths shown in Fig. 2, at point 1 with a time delay of 11τ and an amplitude of

$$\mathcal{F}_0 \left(\frac{R_1}{R_1 + R_2} \right) (1 - r_0) r_0^2 r_1^2 t_1^2 t_2^2 r_2^3 \quad \text{for each of the four waves.}$$

The pulse summation method is a time domain method which is based upon a systematic enumeration of equivalent paths within a structure [1]. The wave-mode coordinate method, on the other hand, is a frequency domain

method which gives no explicit consideration to the existence of equivalent paths. The numerical constants which appear in the wave-mode coordinate solution, and which in fact account for the existence of equivalent paths, appear naturally in the wave-mode coordinate method as a part of the process of Fourier inversion [2].

CONCLUSIONS AND RECOMMENDATIONS

In this investigation, nondispersive pulse propagation in a one-dimensional lattice structure consisting of two segments is analyzed using the pulse summation method and the wave-mode coordinate method. It is shown that the two methods give identical results, and that both methods account for the existence of equivalent paths in the lattice structure.

Both the pulse summation method and the wave-mode coordinate method may be extended to nondispersive pulse propagation in one-dimensional lattice structures consisting of an arbitrary number of segments. Such an extension of the pulse summation method is considered in [1].

Using the general procedures described in [2], the extension of the wave-mode coordinate method to the analysis of nondispersive pulse propagation in two and three-dimensional lattice structures presents no major conceptual difficulties. The extension of the pulse summation method to two and three-dimensional structures seems feasible, but has not yet been accomplished. The exploration of equivalent paths in two and three-dimensional lattice structures may prove to be very interesting.

The problem of dispersive pulse propagation in lattice structures may also be analyzed by the wave-mode coordinate method described in [2] with no major conceptual difficulties. Computational difficulties are expected, however, for complex structures. The extension of the pulse summation method to dispersive pulse propagation does appear to present major conceptual difficulties. In the dispersive problem, pulses do not maintain their shape as they propagate, and it is not clear how to include the effects of dispersion into the pulse summation method.

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- [2] J.H. Williams, Jr. and R.J. Nagem, "Wave-Mode Coordinates and Scattering Matrices for Dynamic Analysis of Large Space Structures," AFOSR Technical Report, October 1986.

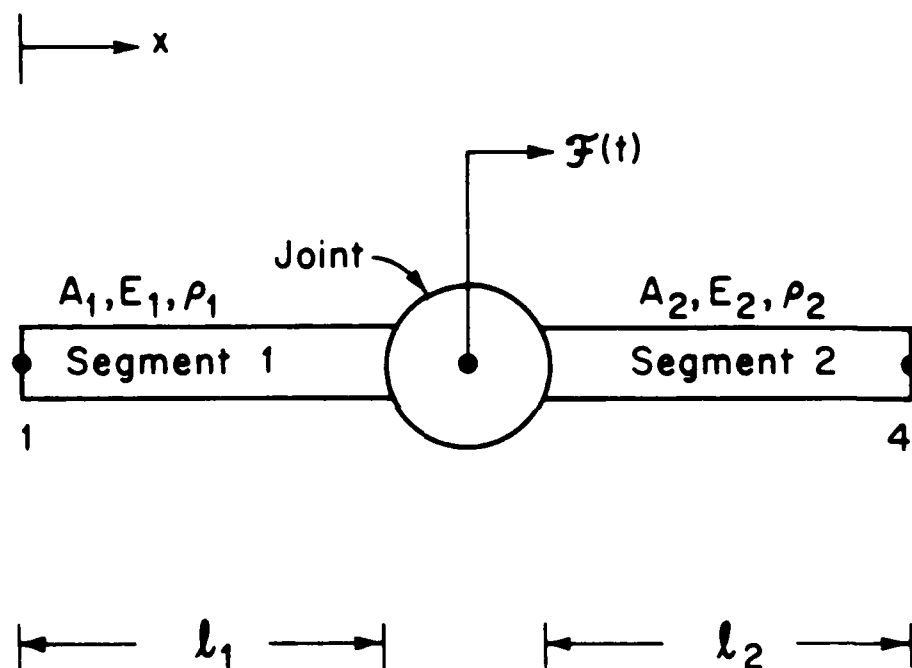


Fig. 1 One-dimensional lattice structure.

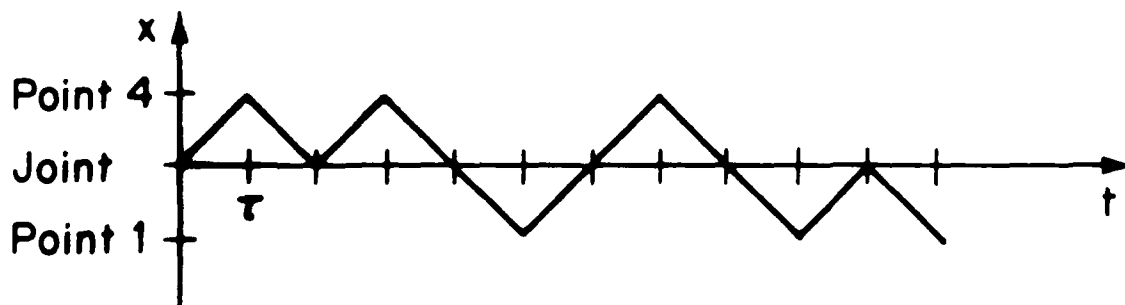
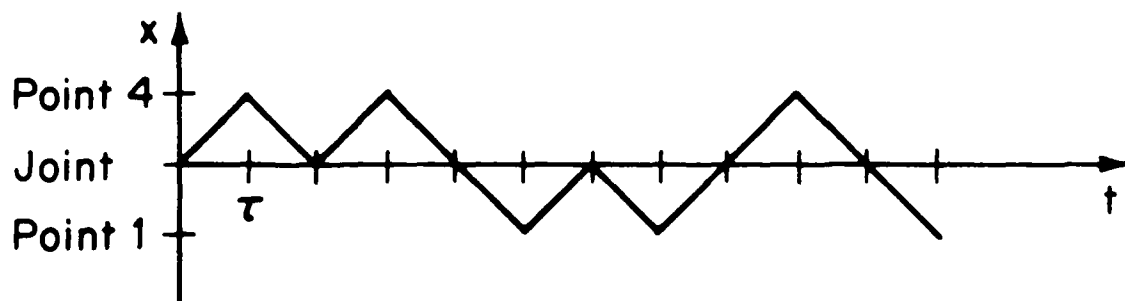
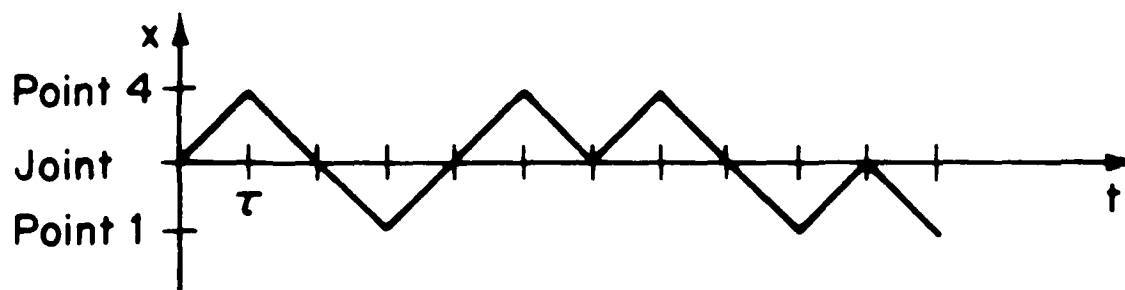
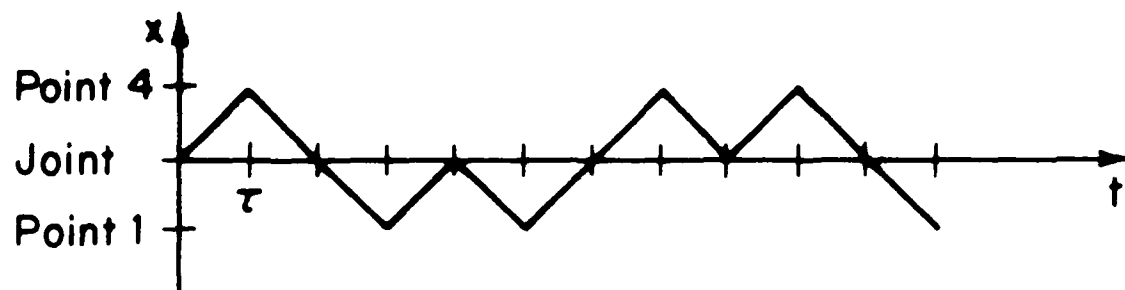


Fig. 2 Equivalent paths in one-dimensional lattice structure.

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